Quantum Corrections to the Ground State Properties of Dilute Bose Liquids[1]

Paulo F. Bedaque, Gautam Rupak
Nuclear Scince Division, Lawrence-Berkeley Laboratory

Aurel Bulgac
Department of Physics, University of Washington

It was recently shown that an entirely new class of quantum liquids with widely tunable propertie could be manufactured from bosons (boselets), fermions (fermilets) and their mixtures (ferbolets) by controlling their interaction properties by the means of a Feshbach resonance. We extend the previous meanfield analysis of these quantum liquids by computing the lowest order quantum corrections to the ground state energy and the depletion of the Bose–Einstein condensate and by estimating higher order corrections as well. We show that the quantum corrections are relatively small and controlled by the diluteness parameter $\sqrt{n|a|^3} \ll 1$, even tough strictly speaking in this case there is no low density expansion.

We find the energy of the system to be given by

$$\mathcal{E} = \frac{g_2\phi^4}{2} + \frac{g_3\phi^6}{6} + \frac{8\hbar^2}{15\pi^2 m}\left(\frac{m\chi}{\hbar^2}\right)^{5/2} + \mu(n-\phi^2).$$

where ϕ is the condensate, $\chi = g_2\phi^2 + g_3\phi^4$ and the couplings are defined by

$$\begin{aligned} g_2 &= \frac{4\pi\hbar^2 a}{m}, \\ g_3 &= \frac{12\pi\hbar^2 a^4}{m}\left[d_1 + d_2\tan\left(s_0 \ln\frac{a}{a_0} + \frac{\pi}{2}\right)\right] \\ &= \frac{6\pi\hbar^2 a^4}{m}\Upsilon \end{aligned}$$

where a is the two–body scattering length, $s_0 \approx 1.00624$ and d_1 and $d_2 < 0$ are universal constants, whose numerical values are known. a_0 is the value of the two–body scattering length for which a three–body bound state has exactly zero energy. Unlike d_1 and d_2, the parameter a_0 is system dependent and is also a genuine three–body characteristic.

We can see that, by adjusting a by the use of a Feshbach resonance, one can reach a state that is self-bound (liquid), with arbitrarily small density.

Corrections to the result above can be estimated and shown to be parametrically small. A generic diagram contributing to the energy density (that is, without external legs) containing I propagators, L loops and n_i vertices with i legs and can be estimated by

$$\begin{aligned} \text{Energy Diagram} &\sim \left(\frac{m}{Q^2}\right)^I \left(\sqrt{\frac{\chi}{m}}QQ^3\right)^L \\ &\times \left(\frac{\chi}{n^{1/2}}\right)^{n_3}\left(\frac{\chi}{n}\right)^{n_4}\left(\frac{\chi}{n^{3/2}}\right)^{n_5}\left(\frac{\chi}{n^2}\right)^{n_6} \\ &\sim m^{3L/2}\chi^{-I+5L/2+n_3+n_3+n_5+n_6} \\ &\times n^{-n_3/2-n_4-3n_5/2-2n_6} \end{aligned}$$

where $Q \sim \sqrt{m\chi}$ is the typical loop momentum. Thus, the result of the mean field analysis predicting the existence of a new dilute quantum liquid is correct.

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